MATHEMATICS (MATH)

MATH 501: Real Analysis

3 Credits

Legesgue measure theory. Measurable sets and measurable functions. Legesgue integration, convergence theorems. Lp spaces. Decomposition and differentiation of measures. Convolutions. The Fourier transform. MATH 501 Real Analysis I (3) This course develops Lebesgue measure and integration theory. This is a centerpiece of modern analysis, providing a key tool in many areas of pure and applied mathematics. The course covers the following topics: Lebesgue measure theory, measurable sets and measurable functions, Lebesgue integration, convergence theorems, Lp spaces, decomposition and differentiation of measures, convolutions, the Fourier transform.

Prerequisite: MATH 404

MATH 502: Complex Analysis

3 Credits

Complex numbers. Holomorphic functions. Cauchy's theorem. Meromorphic functions. Laurent expansions, residue calculus. Conformal maps, topology of the plane. MATH 502 Complex Analysis (3) This course is devoted to the analysis of differentiable functions of a complex variable. This is a central topic in pure mathematics, as well as a vital computational tool. The course covers the following topics: complex numbers, holomorphic functions, Cauchy's theorem, meromorphic functions, Laurent expansions, residue calculus, conformal maps, topology of the plane.

Prerequisite: MATH 501

MATH 503: Functional Analysis

3 Credits

Banach spaces and Hilbert spaces. Dual spaces. Linear operators. Distributors, weak derivatives. Sovolev spaces. Applications to linear differential equations. MATH 503 Functional Analysis (3) This course develops the theory needed to treat linear integral and differential equations, within the framework of infinite-dimensional linear algebra. Applications to some classical equations are presented. The course covers the following topics: Banach and Hilbert spaces, dual spaces, linear operators, distributions, weak derivatives, Sobolev spaces, applications to linear differential equations.

Prerequisite: MATH 501

MATH 504: Analysis in Euclidean Space

3 Credits

The Fourier transform in L1 and L2 and applications, interpolation of operators, Riesz and Marcinkiewics theorems, singular integral operators.

Prerequisite: MATH 502

MATH 505: Mathematical Fluid Mechanics

3 Credits

Kinematics, balance laws, constitutive equations; ideal fluids, viscous flows, boundary layers, lubrication; gas dynamics.

Prerequisite: MATH 402 or MATH 404

MATH 506: Ergodic Theory

3 Credits

Measure-preserving transformations and flows, ergodic theorems, ergodicity, mixing, weak mixing, spectral invariants, measurable partitions, entropy, ornstein isomorphism theory.

Prerequisite: MATH 502

MATH 507: Dynamical Systems I

3 Credits

Fundamental concepts; extensive survey of examples; equivalence and classification of dynamical systems, principal classes of asymptotic invariants, circle maps.

Prerequisite: MATH 502

MATH 508: Dynamical Systems II

3 Credits

Hyperbolic theory; stable manifolds, hyperbolic sets, attractors, Anosov systems, shadowing, structural stability, entropy, pressure, Lyapunov characteristic exponents and non-uniform hyperbolicity.

Prerequisite: MATH 507

MATH 511: Ordinary Differential Equations I

3 Credits

Existence and uniqueness, linear systems, series methods, Poincare-Bendixson theory, stability.

Prerequisite: MATH 411 or MATH 412

MATH 513: Partial Differential Equations I

3 Credits

First order equations, the Cauchy problem, Cauchy-Kowalevski theorem, Laplace equation, wave equation, heat equation.

Prerequisite: MATH 411 or MATH 412

MATH 514: Partial Differential Equations II

3 Credits

Sobolev spaces and Elliptic boundary value problems, Schauder estimates. Quasilinear symmetric hyperbolic systems, conservation laws.

Prerequisite: MATH 502 , MATH 513

MATH 515: Classical Mechanics and Variational Methods

3 Credits

Introduction to the calculus of variations, variational formulation of Lagrangian mechanics, symmetry in mechanical systems, Legendre transformation, Hamiltonian mechanics, completely integrable systems.

Prerequisite: MATH 401 , MATH 411 , or MATH 412

MATH 516: Stochastic Processes

3 Credits

Markov chains; generating functions; limit theorems; continuous time and renewal processes; martingales, submartingales, and supermartingales; diffusion processes; applications.

Prerequisite: MATH 416

MATH 517: Probability Theory

3 Credits

Measure theoretic foundation of probability, distribution functions and laws, types of convergence, central limit problem, conditional probability, special topics.

Prerequisite: MATH 403
Cross-listed with: STAT 517

MATH 518: Probability Theory

3 Credits

Measure theoretic foundation of probability, distribution functions and laws, types of convergence, central limit problem, conditional probability, special topics.

Prerequisite: STAT 517
Cross-listed with: STAT 518

MATH 519: Topics in Stochastic Processes

3 Credits

Selected topics in stochastic processes, including Markov and Wiener processes; stochastic integrals, optimization, and control; optimal filtering.

Prerequisite: STAT 516 , STAT 517
Cross-listed with: STAT 519

MATH 523: Numerical Analysis I

3 Credits

Approximation and interpolation, numerical quadrature, direct methods of numerical linear algebra, numerical solutions of nonlinear systems and optimization. MATH 523 Numerical Analysis I (3) 1. Approximation and interpolation. Weierstrass theorem, Bernstein polynomials, Jackson theorems, Lagrange interpolation, least squares approximation, orthogonal polynomials, piecewise Lagrange and Hermite interpolation, spline interpolation, the Fast Fourier Transform.2. Numerical quadrature. Newton-Cotes rules, Peano Kernel Theorem, Euler-Maclaurin expansion, Romberg integration, Gaussian quadrature, adaptive quadrature.3. Direct methods of numerical linear algebra. Gaussian elimination with pivoting, backward error analysis, conditioning of linear systems.4. Numerical solution of nonlinear systems and optimization. One-point iterations, Newton's and quasi-Newton's method, Broyden's method, unconstrained optimization, line-search methods.

Prerequisite: MATH 456

MATH 524: Numerical Linear Algebra

3 Credits

Matrix decompositions. Direct method of numerical linear algebra. Eigenvalue computations. Iterative methods. MATH 524 Numerical Linear Algebra (3) This course provides a graduate level foundation in numerical linear algebra. It covers the mathematical theory behind numerical algorithms for the solution of linear systems of equations and eigenvalue problems. Specific topics include: matrix decompositions, direct methods of numerical linear algebra, eigenvalue computations, iterative methods.

Prerequisite: MATH 535

MATH 527: Topology

3 Credits

This course provides an overview of the fundamental concepts of Geometric and Algebraic Topology and presents examples of calculations of principal topological invariants. It starts with review of general topology and covers the following topics: fundamental group, homology theories, index theory, CW complexes, and examples of calculations.

Prerequisite: MATH 429

MATH 528: Differentiable Manifolds

3 Credits

Smooth manifolds, smooth maps, Sard's theorem. The tangent bundle, vector fields, differential forms, integration on manifolds. Foliations. De Rham cohomology; simple applications. Lie groups, smooth actions, quotient spaces, examples. MATH 528 Differentiable Manifolds (3) This course covers the foundations of differential geometry, developing the theory of differentiation and integration on manifolds. It provides tools for the study of nonlinear problems, combining techniques in analysis and geometry. Concepts and tools from differential geometry have found wide use in different areas of mathematics, including nonlinear differential equations, control and optimization problems, and numerical analysis. The goal is to cover the most important techniques of differential geometry in a concise way. The course will appeal not only to students who plan to do research in geometry, but also to those interested in analysis, or applied and computational mathematics, as well. It covers the following topics: smooth manifolds, smooth maps, Sard’s theorem, the tangent bundle, vector fields, differential forms, integration on manifolds, foliations, de Rham cohomology, Lie groups, smooth actions, quotient spaces, examples.

Prerequisite: MATH 527

MATH 529: Algebraic Topology

3 Credits

Manifolds, Poincare duality, vector bundles, Thom isomorphism, characteristic classes, classifying spaces for vector bundles, discussion of bordism, as time allows.

Prerequisite: MATH 528

MATH 530: Differential Geometry

3 Credits

Distributions and Frobenius theorem, curvature of curves and surfaces, Riemannian geometry, connections, curvature, Gauss-Bonnet theorem, geodesic and completeness.

Prerequisite: MATH 528

MATH 533: Lie Theory I

3 Credits

Lie groups, lie algebras, exponential mappings, subgroups, subalgebras, simply connected groups, adjoint representation, semisimple groups, infinitesimal theory, Cartan's criterion.

Prerequisite: MATH 528

MATH 534: Lie Theory II

3 Credits

Representations of compact lie groups and semisimple lie algebras, characters, orthogonality, Peter-Weyl theorem, Cartan-Weyl highest weight theory.

Prerequisite: MATH 533

MATH 535: Linear Algebra and Its Applications

3 Credits

Vector spaces. Linear transformations. Inner products and quadratic forms. Theory of endomorphisms of a finite-dimensional vector space. Orthogonal bases, spectral theorem and applications.

Prerequisite: MATH 436

MATH 536: Abstract Algebra

3 Credits

Groups. Sylow's theorems. Rings. Ideals, unique factorization domains. Finitely generated modules. Fields. Algebraic and transcendental field extensions, Galois theory. MATH 536 Abstract Algebra (3) This course covers fundamental concepts, needed toward the study of advanced areas in abstract algebra. The course covers the following topics: groups, Sylow's theorems, rings, ideals, unique factorization domains, finitely generated modules, fields, algebraic and transcendental field extensions, Galois theory.

Prerequisite: MATH 535

MATH 537: Field Theory

3 Credits

Finite and infinite algebraic extensions; cyclotomic fields; transcendental extensions; bases of transcendence, Luroth's theorem, ordered fields, valuations; formally real fields.

Prerequisite: MATH 536

MATH 538: Commutative Algebra

3 Credits

Topics selected from Noetherian rings and modules, primary decompositions, Dedekind domains and ideal theory, other special types of commutative rings or fields.

Prerequisite: MATH 536

MATH 547: Algebraic Geometry I

3 Credits

Affine and projective algebraic varieties; Zariski topology; Hilbert Nullstellensatz; regular functions and maps; birationality; smooth varieties normalization; dimension.

Prerequisite: MATH 536

MATH 548: Algebraic Geometry II

3 Credits

Topics may include algebraic curves, Riemann-Roch theorem, linear systems and divisors, intersectino theory, schemes, sheaf cohomology, algebraic groups.

Prerequisite: MATH 547

MATH 550: Numerical Linear Algebra

3 Credits

Solution of linear systems, sparse matrix techniques, linear least squares, singular value decomposition, numerical computation of eigenvalues and eigenvectors.

Prerequisite: MATH 441 or MATH 456
Cross-listed with: CSE 550

MATH 551: Numerical Solution of Ordinary Differential Equations

3 Credits

Methods for initial value and boundary value problems; convergence and stability analysis, automatic error control, stiff systems, boundary value problems.

Prerequisite: MATH 451 or MATH 456
Cross-listed with: CSE 551

MATH 552: Numerical Solution Of Partial Differential Equations

3 Credits

Finite difference methods for elliptic, parabolic, and hyperbolic differential equations; solutions techniques for discretized systems; finite element methods for elliptic problems.

Prerequisite: MATH 402 or MATH 404 ; MATH 451 or MATH 456
Cross-listed with: CSE 552

MATH 553: Introduction to Approximation Theory

3 Credits

Interpolation; remainder theory; approximation of functions; error analysis; orthogonal polynomials; approximation of linear functionals; functional analysis applied to numerical analysis.

Prerequisite: MATH 401 , 3 credits in Computer Science and Engineering

MATH 555: Numerical Optimization Techniques

3 Credits

Unconstrained and constrained optimization methods, linear and quadratic programming, software issues, ellipsoid and Karmarkar's algorithm, global optimization, parallelism in optimization.

Prerequisite: CMPSC456
Cross-listed with: CSE 555

MATH 556: Finite Element Methods

3 Credits

Sobolev spaces, variational formulations of boundary value problems; piecewise polynomial approximation theory, convergence and stability, special methods and applications.

Prerequisite: MATH 502 , MATH 552
Cross-listed with: CSE 556

MATH 557: Mathematical Logic

3 Credits

The predicate calculus; completeness and compactness; Godel's first and second incompleteness theorems; introduction to model theory; introduction to proof theory.

Prerequisite: MATH 435 or MATH 457

MATH 558: Foundations of Mathematics I

3 Credits

Decidability of the real numbers; computability; undecidability of the natural numbers; models of set theory; axiom of choice; continuum hypothesis.

Prerequisite: any 400 level math course

MATH 559: Recursion Theory I

3 Credits

Recursive functions; degrees of unsolvability; hyperarithmetic theory; applications to Borel combinatorics. Computational complexity. Combinatory logic and the Lambda calculus.

Prerequisite: MATH 557 , or MATH 558

MATH 561: Set Theory I

3 Credits

Models of set theory. Inner models, forcing, large cardinals, determinacy. Descriptive set theory. Applications to analysis.

Prerequisite: MATH 557 or MATH 558

MATH 565: Foundations of Mathematics II

3 Credits

Subsystems of second order arithmetic; set existence axioms; reverse mathematics; foundations of analysis and algebra.

Prerequisite: MATH 557 , MATH 558

MATH 567: Number Theory I

3 Credits

Congruences, quadratic residues, arithmetic functions, partitions, classical multiplicative ideal theory, valuations and p-adic numbers; primes in arithmetic progression, distribution of primes.

Prerequisite: MATH 421

MATH 568: Number Theory II

3 Credits

Congruences, quadratic residues, arithmetic functions, partitions, classical multiplicative ideal theory, valuations and p-adic numbers; primes in arithmetic progression, distribution of primes.

Prerequisite: MATH 421

MATH 569: Algebraic Number Theory I

3 Credits

Dedekind rings; cyclotomic and Kummer extensions; valuations; ramification, decomposition, inertial groups; Galois extensions; locally compact groups of number theory.

Prerequisite: MATH 536 , MATH 568

MATH 570: Algebraic Number Theory II

3 Credits

Topics chosen from class field theroy; integral quadratic forms; algebraic and arithmetic groups; algebraic function of one variable.

Prerequisite: MATH 569

MATH 571: Analytic Number Theory I

3 Credits

Improvements of the prime number theorem, L-functions and class numbers, asymptotic and arithmetic properties of coefficients of modular forms.

Prerequisite: MATH 421

MATH 572: Analytic Number Theory II

3 Credits

Distribution of primes, analytic number theory in algebraic number fields, transcendental numbers, advanced theory of partitions.

Prerequisite: MATH 571

MATH 573: Partitions

3 Credits

This course provides an overview of theory of partitions. This course focuses on the partition function p(n) and its number theoretic behavior and combinatorial structure, along with related problems. To achieve the main goal, generating function theory, basic hypergeometric series, q-series, and some related combinatorial theory are discussed. Building on these relevant theories, students will be able to understand and prove the arithmetic properties of p(n) and related identities including the Rogers-Ramanujan identities. Students will also be able to recognize and apply the same methods to similar functions and identities.

MATH 574: Topics in Logic and Foundations

3-6 Credits/Maximum of 6

Topics in mathematical logic and the foundations of mathematics.

Prerequisite: MATH 558

MATH 577: Stochastic Systems for Science and Engineering

3 Credits

The course develops the theory of stochastic processes and linear and nonlinear stochastic differential equations for applications to science and engineering.

Prerequisite: MATH 414 or MATH 418 ; M E 550 or MATH 501
Cross-listed with: ME 577

MATH 578: Theory and Applications of Wavelets

3 Credits

Theory and physical interpretation of continuous and discrete wavelet transforms for applications in different engineering disciplines.

Prerequisite: M E 550 or MATH 501
Cross-listed with: ME 578

MATH 580: Introduction to Applied Mathematics I

3 Credits

A graduate course of fundamental techniques including tensor, ordinary and partial differential equations, and linear transforms.

Prerequisite: Basic knowledge of linear algebra, vector calculus and ODE, MATH 405

MATH 581: Introduction to Applied Mathematics II

3 Credits

A graduate course of fundamental techniques including Ordinary, Partial, and Stochastic Differential Equations, Wavelet Analysis, and Perturbation Theory.

MATH 582: Introduction to C* Algebra Theory

3 Credits

Basic properties of C* algebras, representation theory, group C* algebras and crossed products, tensor products, nuclearity and exactness.

Prerequisite: MATH 503

MATH 583: Introduction to K-Theory

3 Credits

K-theory groups for compact spaces and C*-algebras. Long exact sequences, Bott periodicity, index theory and the Pimsner-Voiculescu theorem.

Prerequisite: MATH 503

MATH 584: Introduction to von Neumann Algebras

3 Credits

Comparison of projections, traces, tensor products, ITPFI factors and crossed products, the Jones index, modular theory, free probability. MATH 584 Introduction to von Neumann Algebras (3) A concise introduction to von Neumann algebra theory, beginning with the basic definitions and proceeding through modular theory. The currently important subjects of index theory and free probability theory will be introduced.

Prerequisite: MATH 503

MATH 585: Topics in Mathematical Modeling

3 Credits

Introduction to mathematical modeling, covering the basic modeling and common mathematical techniques for problems from physical, biological and social sciences.

Prerequisite: MATH 403 , MATH 411 , and MATH 412

MATH 588: Complexity in Computer Algebra

3 Credits

Complexity of integer multiplication, polynomial multiplication, fast Fourier transform, division, calculating the greatest common divisor of polynomials.

Cross-Listed

MATH 596: Individual Studies

1-9 Credits/Maximum of 9

Creative projects, including nonthesis research, which are supervised on an individual basis and which fall outside the scope of formal courses.

MATH 597: Special Topics

1-9 Credits/Maximum of 9

Formal courses given on a topical or special interest subject which may be offered infrequently; several different topics may be taught in one year or term.

MATH 600: Thesis Research

1-15 Credits/Maximum of 999

No description.

MATH 601: Ph.D. Dissertation Full-Time

0 Credits/Maximum of 999

No description.

MATH 610: Thesis Research Off Campus

1-15 Credits/Maximum of 999

No description.